Indefinite Integration

-Integration works around the idea of working the derivative "backwards" called **antidifferentiation**.

-A function F is an <u>antiderivative</u> of f on an interval I if F'(x) = f(x) for all x in I.

$$F(x) = x^3$$
 becomes $\frac{d}{dx}[x^3] = 3x^2$

-A differential equation in x and y is an equation that involves x and y and the derivatives of y

-Ex

$$y' = 3x \qquad \qquad y' = x^2 + 1$$

Example

-Find the general solution to the differential equation y'=2.

$$y = 2x + C$$

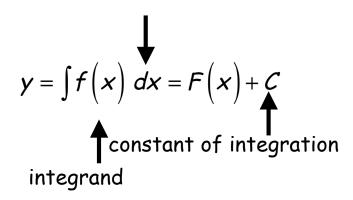
Notation

-When solving the form $\frac{dy}{dx} = f(x)$ it is sometimes convenient to write in the form

$$dy = f(x)dx$$

-The operation of finding all solutions of this equation is called $\underline{\text{antidifferentiation}}$ or $\underline{\text{indefinite integration}}$ and is denoted

variable of integration



Basic Integration Rules

$$\int F'(x)dx = F(x) + C$$

"integration is the "inverse" of differentiation"

$$\frac{d}{dx} \left[\int f(x) dx \right] = f(x)$$

"differentiation is the "inverse" of integration"

Example

Describe the antiderivative of 3x.

$$\int 3x \, dx = 3 \int x \, dx$$

'constant multiple

$$=3\int x^1\,dx$$

$$=3\left(\frac{x^2}{2}\right)+C$$

'Power Rule

$$=\frac{3}{2}x^2+C$$

'Simplify

Steps

-Original Integral \Rightarrow Rewrite \Rightarrow Integrate \Rightarrow Simplify

Examples

$$\int \frac{1}{x^3} dx \Rightarrow x^{-3} dx \Rightarrow \frac{x^{-2}}{-2} + C \Rightarrow -\frac{1}{2x^2} + C$$

$$\int \sqrt{x} dx \Rightarrow \int x^{1/2} dx \Rightarrow \frac{x^{3/2}}{3/2} + C \Rightarrow \frac{2}{3} x^{3/2} + C$$

$$\int 2\sin(x) dx \Rightarrow 2\int \sin(x) dx \Rightarrow 2(-\cos(x)) + C \Rightarrow -2\cos(x) + C$$

Example-Integrating Polynomials

$$\int dx = \int 1 dx$$

$$= x + C$$

$$\int (x + 2) dx = \int x dx + \int 2 dx$$

$$= \frac{x^2}{2} + C_1 + 2x + C_2$$

$$= \frac{x^2}{2} + 2x + C$$

$$\int (3x^4 - 5x^2 + x) dx$$

$$= \frac{3}{5}x^5 - \frac{5}{3}x^3 + \frac{1}{2}x^2 + C$$

Example

$$\int \frac{x+1}{\sqrt{x}} dx$$

$$= \int \left(\frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}}\right) dx$$

$$= \int \left(x^{1/2} + x^{-1/2}\right) dx$$

$$= \frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2} + C$$

$$= \frac{2}{3}x^{3/2} + 2x^{1/2} + C$$

NOTE: Just like differentiation you can't just integrate numerator and denominator.

Example

$$\int \frac{\sin x}{\cos^2 x} dx$$

$$= \int \left(\frac{1}{\cos x}\right) \left(\frac{\sin x}{\cos x}\right) dx$$

$$= \int \sec x \tan x dx$$

$$= \sec x + C$$

Initial Conditions and Particular Solutions

$$y = \int (3x^2 - 1) dx = x^3 - x + C$$
 'General Solution

If you have a value y = F(x) for a value of x called the <u>initial condition</u> you can find the <u>particular solution</u>.

i.e. consider the point (2,4)

$$F(x) = x^3 - x + C$$

$$F(2)=4$$

$$F(2) = 8 - 2 + C = 4$$

$$C = -2$$

So,

$$F(x) = x^3 - x - 2$$
 is a particular solution.

Example

Find the general solution of $F'(x) = \frac{1}{x^2}$, x > 0 and the particular solution that satisfies F(1) = 0.

$$F(x) = \int \frac{1}{x^2} dx$$

$$= \int x^{-2} dx$$

$$=\frac{x^{-1}}{-1}+C$$

$$\frac{-1}{x}+C, x>0$$

$$F(1) = \frac{-1}{1} + C = 0$$

$$C=1$$

$$F(x) = \frac{-1}{x} + 1, x > 0$$

Example

A ball is thrown upward with an initial velocity of 64 ft/sec from an initial height of 80 ft.

Find the position function giving the height s as a function if time t.

When does the ball hit the ground?

Let t = 0 be initial time.

$$s(0) = 80$$
 'Initial Height

$$s'(0) = 64$$
 'Initial velocity

Using -32 ft/sec² as gravity:

$$s''(t) = -32$$

$$s'(t) = \int s''(t)dt = \int -32 dt = -32t + C_1$$

Use initial velocity:

$$s'(0) = 64$$

$$s'(t) = -32t + C_1$$

$$-32(0) + C_1 = 64$$

$$C_1 = 64$$

$$s'(t) = -32t + 64$$

Next by integrating s'(t)

$$s(t) = \int s'(t)dt = \int (-32t + 64) dt$$
$$= -16t^2 + 64t + C_2$$

Using initial height:

$$s(0) = 80$$

$$s(t) = -16t^{2} + 64t + C_{2}$$

$$-16(0)^{2} + 64(0) + C_{2} = 80$$

$$C_{2} = 80$$

$$s(t) = -16t^2 + 64t + 80$$

Solve the position function for where the ball hit the ground.

$$s(t) = -16t^2 + 64t + 80 = 0$$

$$-16(t+1)(t-5)=0$$

$$t = -1.5$$

$$t = 5 \sec$$